Collective oscillations in superconductors revisited

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Abstract

In the recent paper [1] Ohashi and Takada (OT) made statements that in the clean limit considered by us [2], weakly damped collective oscillations in superconductors do not exist due to the Landau damping and their spectrum differs from that obtained in Ref. [2]. In this Comment we would like to note that these statements arise as a result of a misunderstanding of the term "clean" case. OT considered the limit $\omega\tau > 1$, meanwhile Artemenko and Volkov analysed the case $\tau T > 1$, but $\omega\tau < 1$ (!). All these problems were discussed in the review article [3] which was, presumably, unknown to OT.

Collective oscillations (CO) in the superconductors, the search for which had continued since the fifties, were observed in experiments by Carlson and Goldman [4] more than two decades ago. The theoretical explanation for weakly damped CO propagating in superconductors has been suggested in Refs.[2],[5]. Schmid and Schön [5] considered the dirty limit ($\tau T < 1$, τ is the elastic scattering time), and Artemenko and Volkov [2] analysed the "clean" case ($\tau T > 1$). It was shown that in both cases CO has a sound-like spectrum and can exist only near T_c. The general theory of CO for both clean and dirty cases was developed by Ovchinnikov [6] later. The oscillation spectrum can be found from an equation analogous to the continuity equation for, e.g. superfluid component and from an expression for the condensate current. Since the condensate density has a different form in the dirty and clean case, the velocity of CO and the range of their existence are different. All these problems were discussed in detail in the review articles [3],[7],[8]. In spite of this there is still a misunderstnding in the literature about CO. For example, in the recent paper [1] Ohashi and Takada reconsidered the problem of CO (in their view the problem of CO in superconductors is not solved yet) and made the following statements 1. CO in clean superconductors are strongly damped due to the Landau damping, 2. the spectrum of the CO in clean superconductors differs from that obtained in Ref. [2]. In this Comment we would like to note that statement 1. is not new. The importance of the Landau damping and the absence of the weakly damped CO in the limit $\omega\tau > 1$ was noted in Ref. [3]. Correspondingly, the spectrum found in Ref. [1] in this limit does not coincide with the spectrum found in Ref.[2] because the form of the spectrum in this high frequency limit was not presented in Ref. [2] at all (it is of little physical interest as mode is higly damped due to a strong Landau damping). Therefore statement 2. arose due to misunderstanding of the meaning "clean" case. In Ref.[2] "clean" case meant that $\tau T > 1$, but $\omega\tau < 1$ (!). Meanwhile considering "clean" case Ogashi and Takada mean the high fregency limit $\omega\tau > 1$. Below we discuss

briefly the problem of CO in superconductors and how their spectrum depend on the screening by quasiparticles. If the plasma frequency is smaller, than the energy gap Δ (like in layered superconductors), the plasma mode continuosly transforms into the Carlson-Goldman mode as temperature increases [9]. Plasma mode takes place when frequency of dielectric relaxation due to quasiparticle currents is smaller, than the plasma frequency, that is the perturbations of charge densitiy are not screened. Then the superconducting current is compensated by the displacemnt current. At higher temperatures the density of quasiparticles becomes large enough to screen the charge density oscillations totally, and superconducting current is compensated by the current of quasiparticles. In this case oscillations of quasiparticle branch imbalance play an important role. The latter case corresponds to the Carlson-Goldman mode. However, it was shown [10] that in the case of d-pairing intensive relaxation of the branch imbalance due to elastic scattering results in the strong damping of the Carlson-Goldman mode. The statements made above may be illustrated by a transparent and very simple way. Let us consider for simplisity a superconductor with s-pairing. The expression for current density at $\omega \ll 1/\tau$ in the limit $Dk^2 \ll \omega$ has the form

$$j = \frac{c^2}{4\pi\lambda^2} P_s + \sigma_0 \frac{\partial P_s}{\partial t} - \sigma_1 \frac{\partial \mu}{\partial x} + \frac{1}{4\pi}\frac{\partial E}{\partial t}, \tag{1}$$

where the first and the last terms describes the superconducting and the displacement current, respectively. The second and the third terms describe the quasiparticle current. Note that the response to the time derivative of the superconducting momentum, *i. e.* of the gauge-invariant vector potential $\mathbf{P}_s = (1/2)\nabla\chi - (1/c)\mathbf{A}$, and to the gradient of the gauge-invariant scalar potential $\mu = (1/2)(\partial\chi/\partial t) + \Phi$ are described by different generalized conductivities. Thus from the expression for the electric field $\mathbf{E_n} = -\nabla\mu_n - \partial\mathbf{P}_s/\partial t$ one can see that the quasiparticle current can not be discribed by the simple relation $j_{qp} = \sigma E$. At low temperatures, $T \ll \Delta$, in the case of isotropic pairing these conductivities are exponentially small, while near T_c, when $T \gg \Delta$ one gets $\sigma_1 \approx \sigma_N$, $\sigma_0 \approx \sigma_N(1 + \Delta/TJ)$, $J \propto \ln(\Delta/\omega)$. Note that though the difference between σ_0 and σ_1 near T_c is small, it must not be neglected for it determines the upper frequency limit of the range of low damping of the CO. To make the equations complete we need the equation to calculate potential μ related to branch imbalance. Such an equation can be found from the expression for the charge density [3], which can be presented in physically transparent form

$$\frac{\partial\rho}{\partial t} = \gamma\left(\frac{\partial}{\partial t} + \frac{1}{\tau_e}\right)\frac{\kappa^2}{4\pi}\mu + \sigma_1 \frac{\partial^2 P_s}{\partial t \partial x} - \sigma_2 \frac{\partial^2 \mu}{\partial x^2}. \tag{2}$$

Here τ_e is the energy relaxation time. According to this equation variations of charge density are created by variations of the branch imbalance, *i. e.* of the difference between densities of electron-like and of hole-like quasiparticles (the first term in the right-hand side), and by the spatial variations of the quasiparticle flows. Again, the quasiparticle flows are discribed by different "conductivities". At low temperatures $\gamma \approx 1$, "conductivities" being exponentially small. Near T_c $\gamma = \pi\Delta/4\pi$, $\sigma_2 \approx \sigma_N$. The spectrum of eigenmodes can be found equating current j to zero and inserting charge density ρ to the Poisson's equation. The character of the spectrum depends on the relation between plasma frequency $\omega_p = c/(\lambda\sqrt{\varepsilon}$ and frequency of dielectric relaxation $\omega_r = 4\pi\sigma/\varepsilon$. At low temperatures $\omega_p \gg \omega_r$ quasiparticle conductivities

are small. From Poisson's equation we find $\mu = 0$, and from (1) we get $\omega^2 \approx \omega_p^2 - \omega\omega_r$. This equation is strict provided the plasma frequency is smaller, than the gap, however, the result is very similar in the opposite case as well. At high temperatures, near T_c, $\omega_p \ll \omega_r$, and the displacement current can be neglected and Poisson's equation reduces to the neutrality condidtion. Thus, quasiparticles screen the perturbations of the charge density, and quasiparticle current compensates the superconducting current. This makes the mode soft. We find that the region of low damping is limited by conditions

$$\tau_e, \frac{c^2}{\sigma_N \lambda^2} \ll \omega \ll \frac{c^2}{\sigma_N \lambda^2} \frac{T}{\Delta J}.$$

The spectrum in the range of small damping is given by

$$\omega = Vq, \; V^2 = \frac{4\pi \sigma_N c^2}{\kappa^2 \lambda^2}.$$

Using in these equations expressions for $1/\lambda^2$, describing the density of superconducting electrons in the clean and dirty limits, $T_c\tau \gg 1$ and $T_c\tau \gg 1$, respectively, we get the results of papers [2] and [5] for corresponding limits:

$$V = v\sqrt{\frac{7\zeta(3)\Delta}{3\pi^2 T}} \;\; \text{at} \;\; \tau_e, \frac{1}{\tau}\frac{\Delta}{T}^2 \ll \omega \ll \frac{1}{\tau}\frac{\Delta}{T},$$

$$V = v\sqrt{\frac{2\tau\Delta}{3}} \;\; \text{at} \;\; \tau_e, \frac{\Delta^2}{T} \ll \omega \ll \Delta$$

. Note the analogy to the spectrum of phase oscillations in quasi one-dimensional conductors with Charge-Density Wave. Similar to the case of superconductors the Coulomb interactions hardens the Goldstone modes at low temperatures [11], while at high temperatures screening by quasiparticles softens the phason's spectrum making it sound-like both in the case when quasiparticle scattering is neglected [12], and in the collision dominated case [13]. For detailed study of the phason spectrum see [14].

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